

Modified MUSIC Algorithm for CFO Estimation in OFDM System

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Abstract—In orthogonal frequency division multiplexing (OFDM) system carrier frequency offset (CFO) is a noteworthy issue. As slight changes in its value causes critical distortion in system performance. Beforehand, different CFO estimation techniques have been proposed. In this paper CFO is assessed on the premise of subspace based strategies such as multiple signal classification (MUSIC) and modified multiple signal classification (M-MUSIC) algorithms. These algorithms are fundamentally in light of snapshot vector and eigen decomposition of covariance matrix. By introducing exchange matrix(J) in the computation of covariance matrix and the order of matrix depends on number of subcarriers(L). Thus accuracy of MUSIC algorithm is progressed and M-MUSIC come in to existence. Numerical and simulation results exhibit that the M-MUSIC algorithm accomplishes a better estimation around 15dB with HT-6-ray channel over the existing MUSIC algorithm and significantly diminishes the error floor at the receiver.

1. INTRODUCTION

The OFDM system provides better spectral efficiency and minimizes the impact of multipath fading as contrast with single carrier systems.

One of the significant issues with OFDM is that it requires precise frequency synchronization to protect orthogonality between subcarriers, which if decimated will bring about inter-carrier interference (ICI) and results in genuine performance degradation. Frequency offset happens because of mismatch of local oscillator frequency between the transmitter and the receiver or because of doppler movement created by the relative movement between the two conveying terminals. It was perceived that even a little offset error might bring about extreme degradation to the system performance, if not appropriately improved. The insertion of the cyclic prefix(CP) gives high invulnerability against fading channel and guarantees complete elimination of inter symbol interference(ISI) created by multipath propagation.

There are distinctive CFO estimation methodology, namely Data Aided, Non Data Aided (blind). Data Aided presented by Moose[1] and blind proposed by Cimini[2].

Blind technique is not information dependent. It requires cyclic prefix to estimate CFO. It is bandwidth efficient on the expense of moderate transmission speed, estimation accuracy and range which get degraded in debased channel conditions.

The Data-Aided strategy evaluates the frequency offset on the insertion of additional training symbols, for example, preamble and pilot tones. This is done to the detriment of decreasing transmitted information rate by a half esteem atleast. In this method, CFO cancels itself and thus named self cancelation. It yields much preferred execution over that of blind technique as far as transmission speed, estimation precision and range at the expense of bandwidth efficiency.

Various CFO techniques were presented. The major drawback of CP-based estimator is the limited normalized CFO estimation range which is constrained to 0.5. Array based strategy are estimation of parameter via rotational invariance technique (ESPRIT) algorithm[3] and iterative maximum likelihood (IQML) algorithm[4]. They are confined to following area i.e. ESPRIT algorithm typically needs bigger size array $L > 2$ to frame related sub array vector, which is unfeasible to real circumstances, though the IQML algorithm experiencing the convergence problem.

The multiple signal classification (MUSIC) method was proposed in [5],[6] which requires a one dimensional search and its unpredictability depends on search resolution and in which CFO was found to have least power in the null subcarriers. It is feasible for the MUSIC technique to assess the CFO by utilizing a single OFDM symbol. Be that as it may, the CFO estimation precision might be influenced by noise since the MUSIC technique utilizes the observed data acquired from just null subcarriers as opposed to utilizing all sub-carriers. Along these lines, different OFDM symbols are required to diminish the noise impact while assessing the CFO.

The inspiration of this work is to approach the issue of CFO estimation by considering a recently presented subspace-based strategy, M-MUSIC (concerning Forward-Backward Smoothing MUSIC) [7]. From this beginning stage, the principle objective is to construct the simulation scenario,

contrast the acquired results with the broadly known MUSIC and keep on investigating for upgrades of the CFO issue on hold of subspace-based strategies for spectral estimation.

The rest of this paper is composed as follows. Next Section portrays the system model. In Section3 we address the general ICI(inter carrier interference) self cancellation scheme . Section4 characterizes the CFO estimation techniques. Simulation results are presented in Section5, while conclusion is offered in Section6.

2. SYSTEM MODEL

In OFDM frameworks, the wideband frequency selective channel is split into various parallel narrowband flat fading channels which are orthogonal to one another. Fig. 1 demonstrates the OFDM framework which is discrete time and FFT based. Transmitter side include information generator, modulator, IFFT and cyclic prefix adder, while recipient side comprises of cyclic prefix remover, FFT block, demodulator. The information symbol to be transmitted are mapped with BPSK technique and afterward modulated on N subcarriers by utilizing IFFT. Keeping in mind the end goal to maintain a strategic distance from inter symbol interference (ISI), a cyclic prefix of length greater than delay spread is additionally included, by which the impulse response of frequency selective channel get converted into flat fading channel. The complete OFDM symbol is transmitted through a discrete time channel. At the receiver end, the accepted symbol is demodulated and remapped to recover the native symbol.

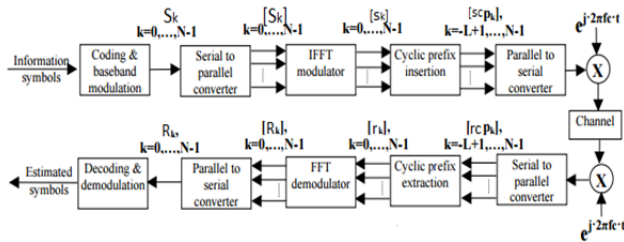


Fig. 1: Block diagram of OFDM system

In OFDM framework, the time domain channel output is expressed as [8]:

$$\mathbf{r}[n]=\mathbf{s}[n]*\mathbf{C}[n]+\mathbf{w}[n] \tag{1}$$

Similarly, representation in matrix form is given as follows:

$$\begin{bmatrix} r_{N-1} \\ r_{N-2} \\ \vdots \\ r_0 \end{bmatrix} = \begin{bmatrix} c_0 & c_1 & \dots & c_\mu & 0 & \dots & 0 \\ 0 & c_0 & \dots & c_{\mu-1} & c_\mu & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & c_0 & \dots & c_{\mu-1} & c_\mu \end{bmatrix} \begin{bmatrix} s_{N-1} \\ s_0 \\ s_{-1} \\ \vdots \\ s_\mu \end{bmatrix} + \begin{bmatrix} w_{N-1} \\ w_{N-2} \\ \vdots \\ w_0 \end{bmatrix}$$

Where; $\mu+1$ is the length of channel impulse response $\mathbf{C}[n]$ and \mathbf{w} is the additive white Gaussian noise vector with zero mean and $No/2$ variance, $n= 0,1,\dots,N-1$. In transmission, OFDM modulator embeds a cyclic prefix(CP) which copies the samples at the end of each OFDM symbols to eliminate the inter symbol interference(ISI). Similarly at the reception side, demodulator removes the CP and decompose the frequency selective channel as a set of mutually orthogonal subcarriers by fast fourier transform(FFT). Introduction of CFO damage the orthogonality among subcarrier and ICI degrade the performance of signal. After including CFO, output response is denoted as:

$$\mathbf{r} = \mathbf{\Lambda}(\epsilon).\mathbf{C}_t.\mathbf{s}_t + \mathbf{w} \tag{2}$$

where, $\mathbf{r}=[r_{N-1},\dots,r_0]^T$ is the CFO affected receiving signal vector with $\mathbf{r}=\mathbf{r}[n]*e^{-j2\pi\epsilon n/N}$

$\mathbf{\Lambda}(\epsilon)=diag\{1, e^{-\frac{j2\pi\epsilon}{N}}, \dots, e^{-\frac{j2\pi\epsilon(N-1)}{N}}\}$ is CFO matrix, \mathbf{C}_t is channel impulse response matrix, $\mathbf{s}_t=[s_{N-1},\dots,s_0]^T$ is channel input vector and \mathbf{w} is the noise vector and After insertion of CP, \mathbf{C}_t is represented as $N \times N$ circular matrix.

Output response in matrix form is represented as,

$$\begin{bmatrix} r_{N-1} \\ r_{N-2} \\ \vdots \\ r_0 \end{bmatrix} = \begin{bmatrix} c_0 & c_1 & \dots & c_\mu & 0 & \dots & 0 \\ 0 & c_0 & \dots & c_{\mu-1} & c_\mu & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & c_0 & \dots & c_{\mu-1} & c_\mu \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ c_2 & c_3 & \dots & c_{\mu-2} & \dots & c_0 & c_1 \\ c_1 & c_2 & \dots & c_{\mu-1} & \dots & 0 & c_0 \end{bmatrix} \begin{bmatrix} s_{N-1} \\ s_{N-2} \\ \vdots \\ s_\mu \end{bmatrix} + \begin{bmatrix} w_{N-1} \\ w_{N-2} \\ \vdots \\ w_0 \end{bmatrix}$$

where Eigen value decomposition of circular matrix \mathbf{C}_t is

$$\mathbf{C}_t = \mathbf{U}^H . \mathbf{C}_f . \mathbf{U} \tag{3}$$

where; \mathbf{U} is $N \times N$ unitary matrix which is given as :

$[\frac{1}{\sqrt{N}} e^{-\frac{j2\pi mn}{N}}]_{m, n=0,1,\dots,N-1}$, \mathbf{U}^H is hermitian matrix of \mathbf{U} , $\mathbf{C}_f = diag\{C_0, \dots, C_{N-1}\}$ is diagonal matrix of

$$C_k = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} c_l e^{-j\frac{2\pi}{N}lk}$$

denoting the k^{th} Eigen value of \mathbf{C}_t . Now, the channel input in time domain is

$$\mathbf{s}_t = \mathbf{U}^H \mathbf{s}_f,$$

in which $\mathbf{S}_f = [S_0, \dots, S_{N-1}]^T$ denotes the frequency domain data vector with S_k being the data symbol at k^{th} subcarrier.

Hence frequency domain representation of received OFDM symbol is

$$\begin{aligned} \mathbf{r} &= \mathbf{U}^* \mathbf{r} = \mathbf{U} \cdot \mathbf{A}(\varepsilon) \cdot \mathbf{C}_t \cdot \mathbf{s}_t + \mathbf{w} \\ \mathbf{r} &= \mathbf{C}_{ICI} \cdot \mathbf{C}_t \cdot \mathbf{s}_t + \mathbf{w} \end{aligned} \quad (4)$$

where, $\mathbf{C}_{ICI} = \mathbf{U} \cdot \mathbf{A}(\varepsilon) \cdot \mathbf{U}^H = [c_{l,m}(\varepsilon)]_{N \times N}$ is ICI matrix with $c_{l,m}(\varepsilon)$ representing the ICI factor between subcarrier l and m .

Hence ICI factor is given as:

$$c_{l,m}(\varepsilon) = \frac{\sin(\pi(l-m+\varepsilon))}{N \sin(\frac{\pi}{N}(l-m+\varepsilon))} e^{-j \frac{(N-1)}{N} \pi(l-m+\varepsilon)} \quad (5)$$

3. GENERAL ICI SELF CANCELLATION SCHEME

By applying ICI self cancellation scheme, Zhao and Haggman[9] proposed an extremely basic and compelling ICI self cancellation procedure. The fundamental thought is to modulate a same information on neighboring subcarriers i.e. when information "a" is modulated on one subcarrier then 'a' will be modulated on nearby subcarrier. The transmitted signal obliged can be given as

$$s(1) = -s(0), \dots, s(N-1) = s(N-2)$$

At the receiver side, the signal is demodulated by linearly combining the adjacent subcarriers by multiplying odd number of subcarriers by '-1'. When $r'(k)$ is received signal on k^{th} subcarrier, then information sequence used for decision symbol will be represented as:

$$r'' = r'(k) - r'(k+1)$$

$$\begin{aligned} r'' &= \sum_{l=0; l=\text{even}}^{N-2} s(l) [-c(l-k-1) + 2c(l-k) - c(l-k+1)] \\ &\quad + \mathbf{w}_k - \mathbf{w}_{k+1} \end{aligned}$$

where, $s'(k)$ and $s'(k+1)$ are the transmitted data symbol at k^{th} and $(k+1)^{\text{th}}$ subcarrier respectively after SC mapping. $r'(k)$ and $r'(k+1)$ are the received data on k^{th} and $(k+1)^{\text{th}}$ subcarrier respectively after performing the FFT operation on the receiver side. r'' is the desired received signal after SC demapping.

Now, n^{th} time sample in equation (2) is rewritten as:

$$r_n = \frac{1}{N} \sum_{k=0}^{N-1} S_k C_k P_k e^{\frac{j2\pi n(k+\varepsilon)}{N}} + w_n \quad (6)$$

where, P_k is processing weight coefficient at subcarrier k for removing the ICI. This process uses the time invariant property of r_n to estimate the CFO. As channel coherence bandwidth is prominent than subcarrier bandwidth, the

channel frequency response remains steady over the band and can be denoted as,

$$C_{kL+l} = C_{kL} \forall l=0, \dots, L-1 \text{ and } k=0, \dots, (N/L)-1$$

By Zhao method, $S_{kL+l} = S_{kL}$ and $P_{kL+l} = P_l, \forall l=0, \dots, L-1$ and $k=0, \dots, (N/L)-1$ where, $P_l = \frac{(L-1)!}{l!(L-1-l)!}$ is the coefficient of the term D^l in the polynomial $(1-D)^{L-1}$ [10], $\sum_{l=0}^{L-1} P_l D^{ln} = (1-D^n)^{L-1}$ and $D = e^{j\frac{2\pi}{N}}$.

Now substitute above identities in equation (6),

$$r_n = \frac{(1-D^n)^{L-1}}{N} \sum_{k=0}^{N/L-1} S_{kL} C_{kL} D^{(kL+\varepsilon)n} + w_n \quad (7)$$

$$\mathbf{r}_n = \mathbf{s}_n + w_n \quad (8)$$

Therefore,

$$s_n = \frac{(1-D^n)^{L-1}}{N} \sum_{k=0}^{N/L-1} S_{kL} C_{kL} D^{(kL+\varepsilon)n} \quad (9)$$

Since the ICI self cancellation scheme utilizes the same weights on each subcarrier band, the composite signal at time $n = n + l \frac{N}{L}$ is,

$$S_{n+l \frac{N}{L}} = \frac{(1-D^{n+l \frac{N}{L}})^{L-1}}{N} \sum_{k=0}^{N/L-1} S_{kL} C_{kL} D^{(kL+\varepsilon)(n+l \frac{N}{L})}$$

where, scaling factor $a_{n,l} = \left(\frac{1-D^{n+l \frac{N}{L}}}{1-D^n} \right)^{L-1}$

To avoid noise enhancement in the equalization process, the time sample selection approach is used in which the values of $a_{n,l}$ are chosen such that,

$$|a_{n,l}^{-1}| \leq 1$$

$$S_{n+l \frac{N}{L}} = a_{n,l} D^{\varepsilon l \frac{N}{L}} S_n$$

Now obtained snapshot vector in time domain is illustrated as:

$$\mathbf{r}_{t,n} = \begin{bmatrix} r_n \\ r_{n+\frac{N}{L}} \\ \vdots \\ r_{n+(L'-1)\frac{N}{L}} \end{bmatrix}$$

where, $L' \leq L$ is stacking size of the snapshot vector and $L' = L-1$ for $L > 2$, $L' = L$ for $L = 2$.

$$\mathbf{r}_{t,n} = \boldsymbol{\Phi} \mathbf{b}(\varepsilon) \mathbf{s}_n + \mathbf{w}_n, n = 0, 1 \dots \left(\frac{N}{L} \right) - 1 \quad (10)$$

where,

$$\boldsymbol{\Phi}_n = \text{diag}\{a_{n,0}, a_{n,1}, \dots, a_{n,L'-1}\}$$

and

$$\mathbf{b}(\epsilon) = \left[1, e^{\frac{j2\pi\epsilon}{L}}, \dots, e^{\frac{j2\pi\epsilon(L-1)}{L}} \right]^T$$

This work also denotes $\mathbf{b}(\epsilon)$ the CFO signature of the OFDM system.

4. CFO ESTIMATION ALGORITHMS

4.1 MUSIC algorithm

Prior to the CFO estimation, scaling effect is removed from equation(10) by multiplying φ^{-1}

$$\hat{\mathbf{r}}_{t,n} = \varphi^{-1} \mathbf{r}_{t,n} = \mathbf{b}(\epsilon)\mathbf{s}_n + \boldsymbol{\omega}_n \quad (11)$$

where,

$$\boldsymbol{\varphi}_n^{-1} = \text{diag} \left\{ a_{n,0}^{-1}, a_{n,1}^{-1}, \dots, a_{n,L-1}^{-1} \right\}$$

represents the equalization matrix and

$$\boldsymbol{\omega}_n = \boldsymbol{\varphi}_n^{-1} * \mathbf{w}_n,$$

is the equalized noise vector.

The MUSIC algorithm[11] is a subspace-based calculation that gauges the parameter of interest by utilizing the eigen structure of obtained array signals. To appraise the CFO, determine the covariance matrix of the obtained signal. Theoretically, the covariance matrix of the obtained signal is given by

$$\mathbf{U}_t = E \{ \hat{\mathbf{r}}_{t,n} \cdot (\hat{\mathbf{r}}_{t,n})^H \} \quad (12)$$

Practically, the covariance matrix is calculated as,

$$\hat{\mathbf{U}}_t = \frac{L}{N} \sum_{n=0}^{L-1} \hat{\mathbf{r}}_{t,n} \cdot (\hat{\mathbf{r}}_{t,n})^H \quad (13)$$

Large number of N/L in above equation is required to get an exact evaluation of \mathbf{U}_t .

Eigen decomposition of \mathbf{U}_t can be analyze as,

$$\hat{\mathbf{U}}_t = [\mathbf{v}_s \quad \mathbf{E}_n] \begin{bmatrix} \eta & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_s^H \\ \mathbf{E}_n^H \end{bmatrix} \quad (14)$$

where, $\mathbf{v}_s = \zeta \frac{\mathbf{b}(\epsilon)}{\|\mathbf{b}(\epsilon)\|}$ and ζ is complex number with

unit magnitude and \mathbf{v}_s contain eigen vector

corresponding to non zero eigen value η .

$$\mathbf{E}_n = [\mathbf{v}_{n,1} \quad \dots \quad \mathbf{v}_{n,L-1}]$$

contains other eigen vectors corresponding to zero eigen values. Since \mathbf{v}_s and \mathbf{E}_n are orthogonal to each other, cost function for the cfo estimation is given by:

$$G(\rho) = \mathbf{b}^H(\rho) \mathbf{E}_n \mathbf{E}_n^H \mathbf{b}(\rho) \quad (15)$$

Cfo is estimated by,

$$\hat{\epsilon} = \text{argmin}_{\rho} G(\rho) \quad (16)$$

where, range of ρ is $\rho \in (-\frac{L}{2}, \frac{L}{2}]$

The pseudo spectra $G^{-1}(\rho)$ is defined by the inverse of cost function $G(\rho)$ in MUSIC algorithm. A key element of this study is its flexible estimation range directly relative to the order of the ICI self-cancellation scheme L . As CFO is estimated in time domain, therefore this approach is known as MUSIC algorithm.

4.2 M-MUSIC algorithm

In this approach, exchange matrix (\mathbf{J}) is introduced in covariance matrix to estimate CFO. Therefore, modified equation for the evaluation of covariance matrix (12) and (13) is given as,

Theoretically,

$$\mathbf{U}_t = \mathbf{J} E \{ \hat{\mathbf{r}}_{t,n} \cdot (\hat{\mathbf{r}}_{t,n})^H \} \mathbf{J} \quad (17)$$

Practically,

$$\hat{\mathbf{U}}_t = \frac{L}{N} \sum_{n=0}^{L-1} \hat{\mathbf{r}}_{t,n} \cdot (\hat{\mathbf{r}}_{t,n})^H + \mathbf{J} \overline{\hat{\mathbf{r}}_{t,n} \cdot (\hat{\mathbf{r}}_{t,n})^H} \mathbf{J} \quad (18)$$

where, \mathbf{J} is the $L \times L$ exchange matrix with all zero except minors diagonal elements which contain ones.

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & & 1 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 1 & \dots & 0 & 0 \\ 1 & 0 & & 0 & 0 \end{bmatrix}$$

Now estimate the CFO using the above equations from (14) to (16).

This approach is also known as forward backward smoothing algorithm[7]. It also provides better estimation accuracy.

5. SIMULATION, RESULTS AND ANALYSIS

This area exhibits the results of computer simulations to analyze the execution of both the approaches. The OFDM symbol size is $N = 256$, the length of CP insertion is $N_{CP} = 15$ and Binary Phase Shift Keying (BPSK) is employed. The exponential sample space channel and HT-6-ray sparse channel are used. The power delay profile (PDP) of the channels are $P_n = \lambda * e^{0.02 * [0,1, \dots, path-1]}$ where $path = T_{max} / T$, T_{max} is channel delay spread and T is sampling time and $P_n = [0.413, 0.2930, 0.145, 0.074, 0.066, 0.008]$, respectively.

Fig.2 and Fig3, compares the pseudospectra of the MUSIC and M-MUSIC algorithm for number of subcarriers that is $L=2$ and 3, respectively with exponential sample space channel.

The exact value of CFO, $\varepsilon = 0.30$ and value of signal to noise ratio (SNR) is 20dB. It demonstrates that both the approaches estimate for $L=2$, However, as value of L increases MUSIC fails to yield the outcome.

Fig.2 and Fig.4, illustrates that on decreasing the value of SNR from 20dB to 18dB for $L=2$ (fixed) and CFO $\varepsilon=0.30$, the execution of MUSIC algorithm is severely influenced while M-MUSIC estimate the CFO precisely.

Fig.5 depicts the pseudospectra of the MUSIC and M-MUSIC algorithm for order $L=3$ with HT-6-ray sparse channel. The exact value of CFO, $\varepsilon = 0.30$ and value of SNR is 15dB. It illustrates that M-MUSIC algorithm with the following channel provides better estimation as compare to previous channel in terms of SNR.

Fig.6 represents the CFO estimation range for M-MUSIC algorithm at $L=2$ and 3, respectively. The value of SNR is 20dB and range of CFO estimated is adjustable in $(-L/2, L/2]$.

Therefore, M-MUSIC provides better estimation than MUSIC algorithm.

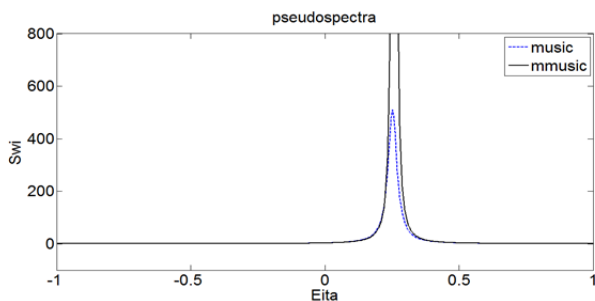


Fig. 2. Pseudo spectra of the MUSIC and M-MUSIC algorithm corresponding to the exponential channel for the order $L=2$ and SNR=20dB respectively.

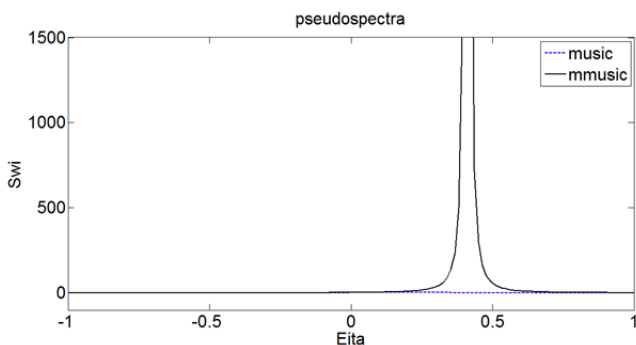


Fig. 3. Pseudo spectra of the MUSIC and M-MUSIC algorithm corresponding to the exponential channel for the order $L=3$ and SNR=20dB respectively.

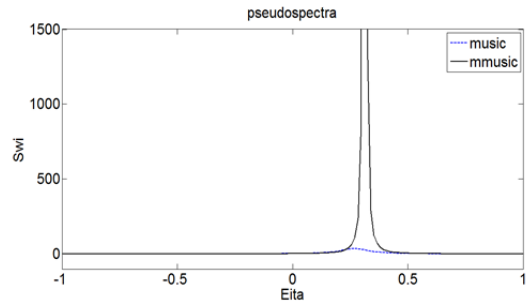


Fig. 4. Pseudo spectra of the MUSIC and M-MUSIC algorithm corresponding to the exponential channel for the order $L=2$ and SNR=18dB respectively.

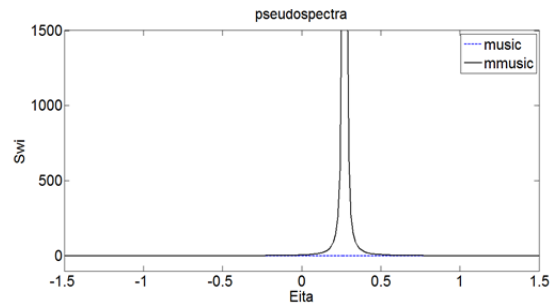


Fig. 5. Pseudo spectra of the MUSIC and M-MUSIC algorithm corresponding to the HT-6-ray channel for the order $L=3$ and SNR=15dB respectively.

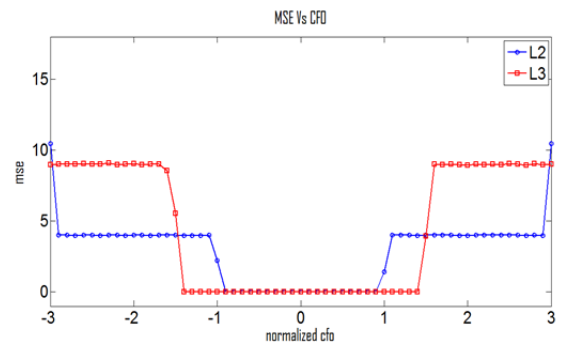


Fig. 6. CFO estimation range of the M-MUSIC algorithm for the order $L=2,3$ and SNR=20dB respectively.

Above results represents that, pseudospectra mainly depends upon number of subcarriers(L) and SNR value. At the point, when SNR is fixed and L is increased or L is fixed and SNR is decreased, the CFO estimation is more accurate for M-MUSIC algorithm as contrast with MUSIC algorithm. On the other hand, simulation result of sparse channel is more viable than sample space channel at low SNR value.

6. CONCLUSION

In this paper, we contemplated the issue of CFO estimation in OFDM system, two pseudospectra, subspace based estimation algorithm were assessed as a first approach to the problem.

Simulation results have shown that M-MUSIC algorithm has a better CFO estimation than MUSIC algorithm over HT-6-ray sparse channel.

An essential perception is that, these subspace based spectral estimation algorithms have demonstrated high affectability to eigen value spread of the covariance matrix, exceptionally for low N/L ratio. This issue might be dealt with in a future work.

We find ahead of time an entire line of investigation in regards to CFO estimation and subspace based algorithms that merits concentrate, for example [3]. The moderately low many sided quality of these strategies makes it exceptionally alluring for a versatile situation as agreeable interchanges, where computational resources and power consumption are basic issues.

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